# A 3D Model Retrieval Method based on Shape Distribution and Curvature

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**Abstract:** Aiming at the problem of low retrieval accuracy when using content-based 3D model retrieval with a single feature, a new 3D model retrieval algorithm is proposed. The method is mainly divided into the following steps: 1) Calculate the shape distribution feature and curvature feature of the 3D models; 2) Calculating a similarity value of each model in the input model and the database according to the shape distribution feature, and using the similarity value to obtain a result set; 3) Using the curvature feature of the 3D models, the similarity between each model in the result set and the input model is calculated; 4) The search result is output according to the similarity calculation result in step 3). The method realizes global and local matching of 3D models, which can effectively improve the matching precision of 3D model retrieval, and has certain application value and reference significance.

#### 1. Introduction

With the development of 3D modeling technology, 3D scanning technology and computer hardware, vivid and realistic 3D models are generated and continuously transmitted through the Internet. People can find a variety of 3D model databases on the network. How to quickly and accurately find the 3D models you need in various model databases and the Internet has become a crucial issue in the field of 3D models.

The key to 3D model retrieval technology is to extract features, and 3D model features are divided into global features and local features according to scope. The global features focus on the overall shape of the 3D model; therefore, global feature can distinguish between large categories. However, when the three-dimensional models are partially similar, it is difficult to achieve local matching using traditional global features. The local features of the 3D model reflect the local characteristics of the 3D model, mainly considering the relationship between the points on the surface of the model and its neighbors. The local feature-based retrieval algorithm can distinguish the nuances between the models, and can effectively improve the retrieval effect of the 3D model and realize the local retrieval of the 3D model.

At present, the research on feature extraction methods of 3D models has been deepened. The shape distribution algorithm proposed by Osada et al. [1] is a simple and effective algorithm for measuring the similarity of 3D entities. The shape context method proposed by Belongie et al. [2] extends form the shape context of two-dimensional images. Darom et al. [3] proposed LD-SIFT (local-depth SIFT), SISI (scale-invariant spin image) features. Wang et al. [4] combines various features and uses sparse subspace clustering to select key features. Finally, a spectral clustering method is used to obtain new combined features. [9] proposed a 3D model retrieval algorithm based on local feature. The curvature  $D=k_1^2+k_2^2$  is defined as the feature descriptor of the 3D model. However, there is no universal 3D model feature extraction method, and each method has a certain range of use. Therefore, it is of great significance and value to improve the existing 3D model retrieval methods or combine new theories to study new 3D model retrieval methods to solve the current difficulties in 3D model retrieval. In this paper, a three-dimensional model retrieval method based on shape distribution and curvature is proposed. Firstly, the shape distribution feature is used to match the model width higher overall similarity as the model to be retrieval, and then the model to be retrieved and the retrieved curvature

feature are calculated. The model output obtained by sorting the similarity from high to low is the final retrieval result.

#### 2. Methodology

## 2.1 Shape Distribution Feature Extraction

The shape distribution method of Osada et al. [1] first calculates a large amount of statistical data, and uses these data to directly form a shape distribution to describe the model features. Therefore, the key is to define the calculation function describing the shape. The literature [5] defines five simple and easy to calculate shape functions:

- A3—the angle formed by any three points on the surface of the 3D model.
- D1—the distance between any point on a 3D model surface to a fixed point, fixed at the geometric center of the 3D model.
  - D2—the distance between any two points on the surface of the 3D model.
- D3—the square root of the area of the triangle formed by any three points on the surface of the 3D model.
  - D4—the cube root of the cube volume formed by any four points on the surface of the 3D model.

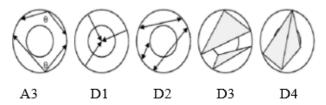


Fig 1. Osada's five shape functions (figure caption)

Osada has done a lot of experiments on A3 distribution, D1 distribution, D2 distribution, D3 distribution and D4 distribution. The analysis results show that the distance between any two vertices is the best as a geometric function [5]. The result also show that the ability of the D2 distribution function to describe the model is more powerful and computationally simple due to the other four shape functions, especially for some simpler three-dimensional models. In this paper, the shape distribution curve of the model to be retrieved and the model to be retrieved is obtained by using the D2 shape distribution function, and the similarity degree of the two curves is measured by the L2 distance function, so as to achieve the overall selection of all the database three-dimensional models.

The specific process of feature extraction of 3D model D2 shape distribution is as follows:

- (1)Divide all the polygons that make up the surface of the 3D mesh model into triangles, form a triangular mesh model and save it for later use.
- (2) Calculate and store the area of all triangular patches of the segmented triangular model. Suppose  $T=(v_1, v_2, v_3)$  represents one of the triangles, and its area can be expressed as

$$S_T = \frac{|(v_2 - v_1) * (v_3 - v_1)|}{2} \tag{1}$$

where,  $v_1$ ,  $v_2$  and  $v_3$  are respectively three vertices in the triangular patch. The total area S of the triangular mesh model is the sum of ST.

(3) First generate a random number between (0, S), and retrieve the data in the array of storage triangle areas established in step (2) (which can be retrieved by binary search). Use formula (2) on the triangular patch to get the coordinates of the qualified feature points:

$$P = (1 - \sqrt{r_1})v_1 + \sqrt{r_1}(1 - r_2)v_2 + \sqrt{r_1}r_2v_3$$
 (2)

where,  $r_1$  and  $r_2$  are random numbers between [0, 1], and P is a feature point.

(4)Let the Euclidean distance between two random points on the surface of the 3D model be  $d_i$ , and calculate the average value of the distance  $d=(d_1,d_2,\cdots,d_i,\cdots,d_n)$ ,  $i=1,2,\cdots,n$ , between all random

points pairs on the surface of the 3D model. The value is equally divided into m (m <n) intervals, each interval width is  $\sum_{i=1}^{n} d_i / mn$ , then calculate Cdj, (j=1,2, ..., m), the number of Euclidean

distances falling within each interval. The horizontal; axis represents the interval distance value, and the vertical axis represents the number of occurrences of a certain distance value, and a D2 shape distribution distance histogram is constructed.

(5) The ratio of the number of occurrences of each interval distance value in step (4) to the total distance forms the feature vector of the model. Then the feature vector of the shape distribution of the model is  $Xd = (Xd_1, Xd_2, \dots, Xd_j, \dots, Xd_m)$ , where  $Xd_i = Cd_i/n$ .

#### 2.2 Curvature Feature Extraction

Literature [6, 7, 8] uses the characteristics of the curvature of the model vertices as a descriptor for model retrieval. For any two-dimensional plane embedded in the Euclidean space R3, there are two kinds of curvatures: average curvature and Gaussian curvature. They are all intrinsic geometric invariants of the surface, which can well describe the degree of bending of a point on the surface, but also have their own defects. Literature [9] defines a new local feature descriptor: curvature. Under the premise of not adding extra calculation, the insensitivity of average curvature to smooth model and the uniform distribution of Gaussian curvature can be overcome at the same time, and the local bending degree of the model can be reflected more realistically.

In summary, this paper uses curvature feature extraction method to match the local features of the retrieval results using the shape distribution features, and obtain a more accurate final retrieval result. The specific 3D model curvature feature extraction process is as follows:

(1)Let the two principal curvatures of a point on the surface of the 3D model be  $k_1$  and  $k_2$ respectively, then the Gaussian curvature can be recorded as  $K=k_1\cdot k_2$ , and the average curvature can be recorded as H=  $(k_1+k_2)/2$ , and the curvature [9] of the 3D model can be written as  $G=k_1^2+k_2^2$ . The curvature G is expended and the Gaussian curvature and the average curvature are added to obtain the formula (3).

$$G=k_1^2+k_2^2=(k_1+k_2)^2-2 k_1\cdot k_2=4H^2-2K$$
(3)

(2) The estimation of Gaussian curvature is based on the method of [10]. The calculation formula

is 
$$K(v_e) = \frac{2\pi - \sum_e \theta_e}{\sum_e S(v_e)}$$
, where  $S(v_e)$  represents the area of the triangle where the vertex  $v_e$  is located,

and  $\theta_e$  represents the degree of the vertex of the triangle where the vertex  $v_e$  is located.

(3) There are many methods of calculating the curvature on the triangular mesh model. In this paper, the average curvature is estimated by the discrete method of Laplace-Beltrami operator [11], and the Laplace-Beltriami operator is introduced. Where  $\Delta$  is the gradient operator, H is the average curvature of the sample points, and n is the normal vector of the sample points. The corresponding discrete mean curvature is:

$$H = \frac{\Delta \cdot \vec{n}}{2} \tag{4}$$

The Laplacian  $\Delta$  is discretized on the triangular mesh surface by the Taubin method. For a point  $v_e$  on a mesh surface, its 1-neighbor point set  $\{v_b,b\in N(e)\}$ , where N(e) represents the set of subscripts of its 1-neighbor vertices, and  $\Delta$  can be expressed as:  $\Delta = \sum_{b \in N(e)} w_{eb} (v_b - v_e), \sum_{b \in N(e)} w_{eb} = 1$ 

$$\Delta = \sum_{b \in N(e)} w_{eb} (v_b - v_e), \sum_{b \in N(e)} w_{eb} = 1$$
(5)

Where, web is a weighting factor, and there are many methods for determining the weighting factor w<sub>eb</sub>. Here we take

$$w_{eb} = \frac{1}{2} \left( \cot \alpha_{eb} + \cot \beta_{eb} \right) \tag{6}$$

Where  $\alpha_{eb}$ ,  $\beta_{eb}$  are  $\angle v_e v_{b-1} v_b$ ,  $\angle v_e v_{b+1} v_b$ , respectively, and points  $v_{b-1}$  and  $v_{b+1}$  are the other vertices of two different triangular patches where the point  $v_e$  and the point  $v_b$  coexist, respectively. Combining formulas (4), (5), (6), the calculation formulas of average curvature are obtained:

$$H = \frac{1}{4} \sum_{b \in N(e)} (\cot \alpha_{eb} - \cot \beta_{eb}) (v_b - v_e) \cdot \vec{n}$$

(4)The curvature of any vertex on the surface of the similarity matching model and the retrieved model is calculated by combining step (3), step (4), and formula (3). Let the curvature value  $G=(G_1,G_2,\cdots,G_f)$  of each model obtained, a total of f. Calculate the average curvature value of the model, and divide the value into q intervals, the length of each interval is  $\sum_{x=1}^f G_x / qf$ , where  $x=1,2,\cdots,f$ , then calculates the number of curvatures falling in each interval, and the model's curvature feature vector is  $Z=(Z_1,Z_2,\cdots,Z_t,\cdots,Z_q)$ , where  $Z_t=CG_t/f$ .

## 2.3 Similarity Calculation

The similarity between 3D models can be calculated by the distance between feature vectors. There are many ways to calculate the distance between two feature vectors, such as L2(Euclidean) distance, L1(Manhattan) distance, Hausdorff distance, etc.

In this paper, the L2 distance is used to calculate the similarity of the shape distribution feature vector of the model. The feature vector of the input model is  $Xd = (Xd_1, Xd_2, \dots, Xd_j, \dots, Xd_m)$ , and the feature vector of the 3D model in the database is  $Yd = (Yd_1, Yd_2, \dots, Yd_j, \dots, Yd_m)$ , then the L2 distance between them is

$$D_{L2}(Xd, Yd) = \sqrt{\sum_{j=1}^{m} (Xd_i, Yd_j)^2} zz$$

The smaller the value of  $D_{L2}$  (Xd, Yd) is, the more similar the two models are, and the greater the similarity value between the retrieved model and the corresponding model in the database. And the similarity calculation of the curvature feature vector of the model is calculated by using the L1 distance. Let the eigenvectors of the two models be  $XG=(XG_1, XG_2, \cdots, XG_t, \cdots, XG_q)$  and  $YG=(YG_1, YG_2, \cdots, YG_t, \cdots, YG_q)$ , and the L1 distance between them is

$$D_{L_1}(XG_t, YG_t) = \sum_{t=1}^{q} |XG_t - YG_t|$$

## 3. Result and Analysis

The computer configuration of this experiment is Intel CPU main frequency is 3.2GHz, memory is 8GB, the test platform is Windows 7 operating system, the code is written by MATLAB 2016a, the data used is extracted from the Princeton Shape Benchmark database. A total of 100 models are selected from 10 categories, including bottles, human head models, human hand models, snowmen and so on. In the experiment, the D2 shape distribution algorithm, the curvature algorithm and the combination of D2 shape distribution and curvature are used to compare the experimental results of the three algorithms.

In the experiment, Precision-Recall curve was used to evaluate the retrieval performance of feature extraction algorithm.

Recovery rate = 
$$\frac{A}{A + C}$$

Precision rate = 
$$\frac{A}{A + B}$$

Where B is the number of models that are looking for errors, C is the number of models that are missing, and A is the number of models that are queried correctly. The Precision-Recall curve obtained from the experiment is shown in the figure 2.

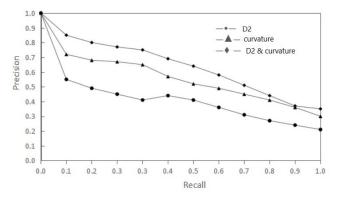


Fig 2. Precision-Recall curve (figure caption)

It can be seen from the figure that compared with the D2 shape distribution or curvature feature algorithm alone, the retrieval algorithm based on shape distribution and curvature feature proposed in this paper has obvious advantages, which can not only improve the precision of model retrieval, but also the retrieval time is reduced to some extent.

### 4. Summary

The algorithm based on shape distribution and curvature feature proposed in this paper improves the traditional method. Combined with global feature matching and local feature matching, it can describe the 3D model better and more comprehensively than using a single feature. In addition, the similarity between the retrieved model and the retrieved model is sorted from high to low in the model database after global retrieval, and partial models with high similarity are selected for local feature matching, which can improve the precision of retrieval and reduce the time cost to some extent. However, since the calculation of the curvature depends on the calculation of the curvature of the discrete mesh surface, the calculation of the curvature on the discrete mesh is susceptible to noise and the calculation is large. Therefore, the more robust and simple curvature calculation is the next improvement direction.

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